Conceptual and Theoretical Models

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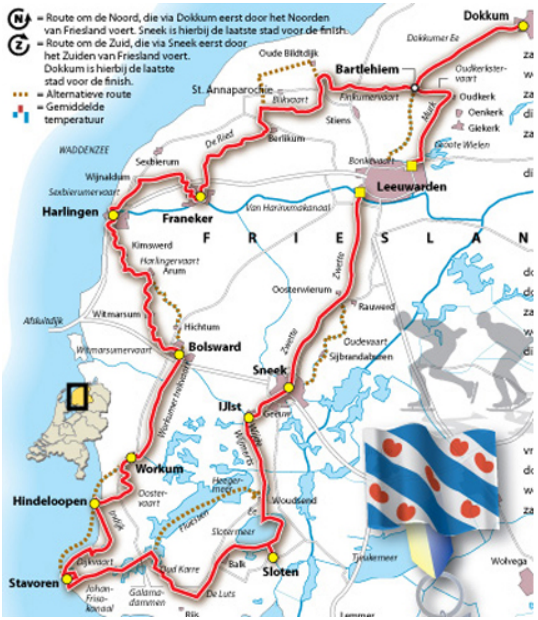
Why atmospheric sciences?



Eleven City Ice Skating Race ~ 200 km

Need extremely cold weather conditions

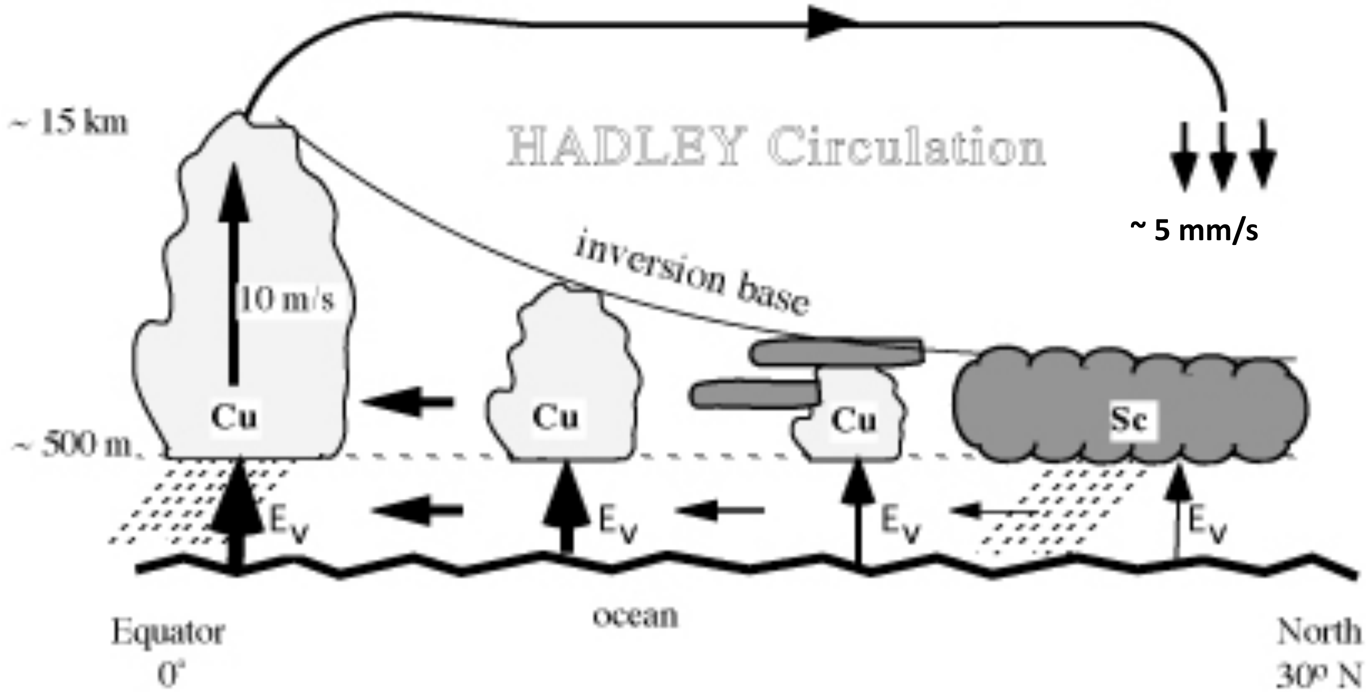
Organized in 1909, 1912, 1917, 1929, 1933, 1940, 1941, 1942, 1947, 1954, 1956, 1963, ...

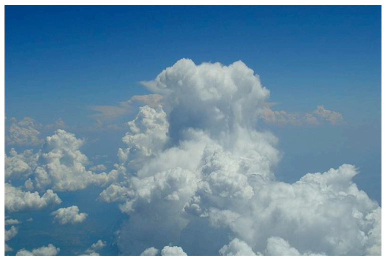
and 1985, 1986, 1997 (and almost in 1984)

1963

-18^{0}C at the start, storm
129 finished (out of 10000)

How do large-scale conditions control cloud amount and cloud type?



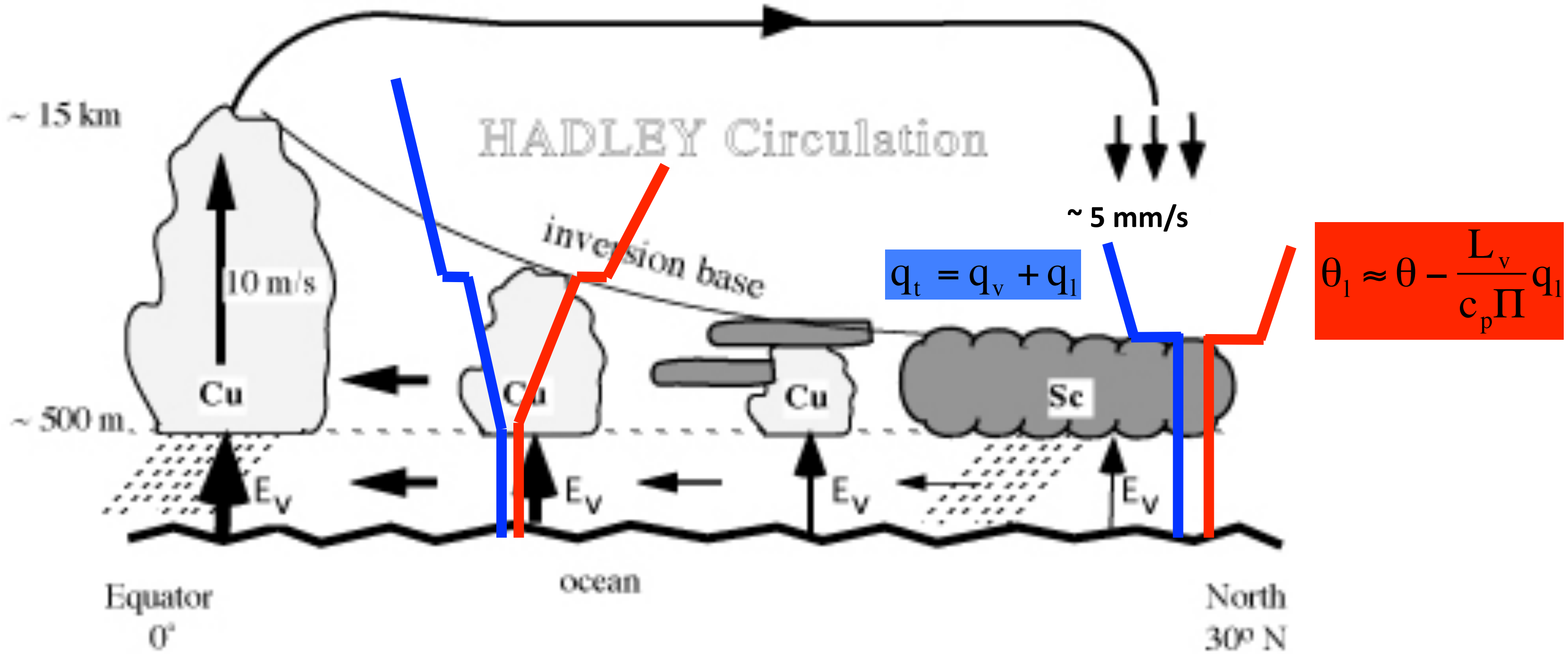


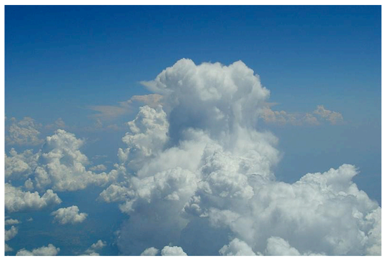
deep convection

shallow cumulus

stratocumulus

How do large-scale conditions control cloud amount and cloud type?





deep convection

shallow cumulus

stratocumulus

Contents

Class 1:

Entrainment and buoyancy fluxes

The 'easy' case: dry convective boundary layer

How to compute the buoyancy flux in the cloud layer

Use fluxes of conserved variables

Mixing diagrams, evaporative cooling, and entrainment

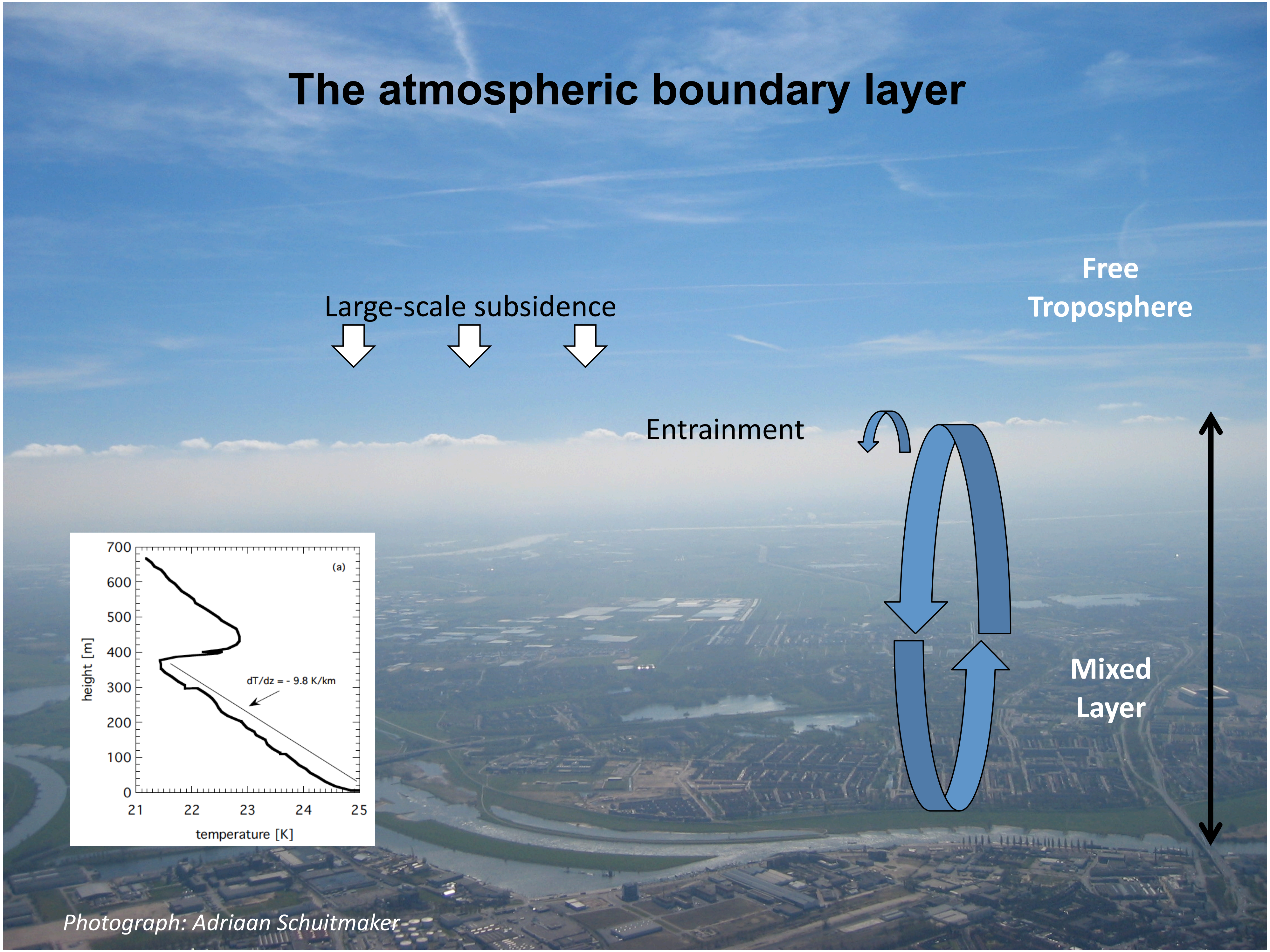
Class 2:

Equilibrium solutions for low clouds

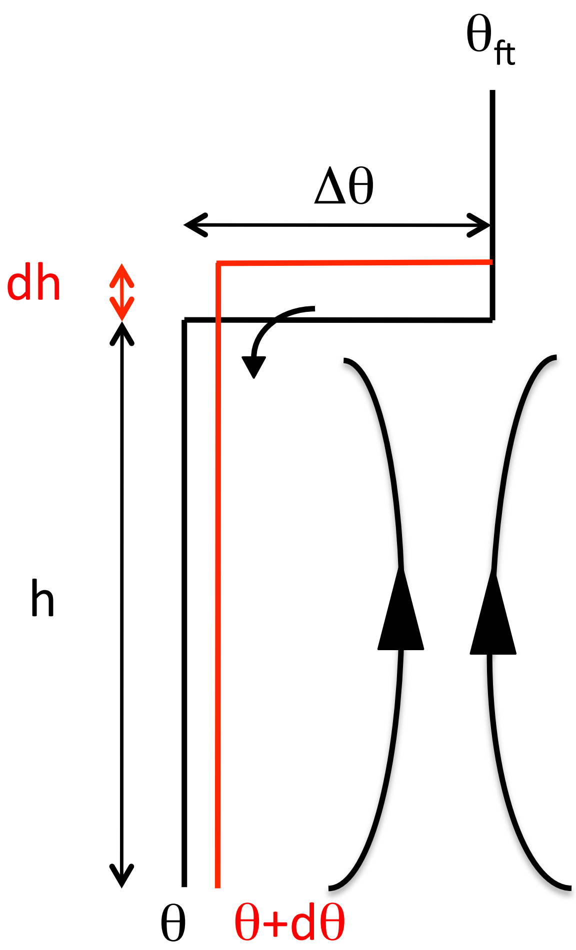
Low cloud transitions

Class 3: (Bjorn et al)

Convective Radiative Equilibrium


The atmospheric boundary layer
Large-scale subsidence
Free Troposphere
Entrainment
Mixed Layer
700
600
500
400
300
200
100
0
height [m]
(a)
dT/dz = - 9.8 K/km
21
22
23
24
25
temperature [K]
Photograph: Adriaan Schuitmaker

Assume eddy entrains air from above and vertically redistributes it



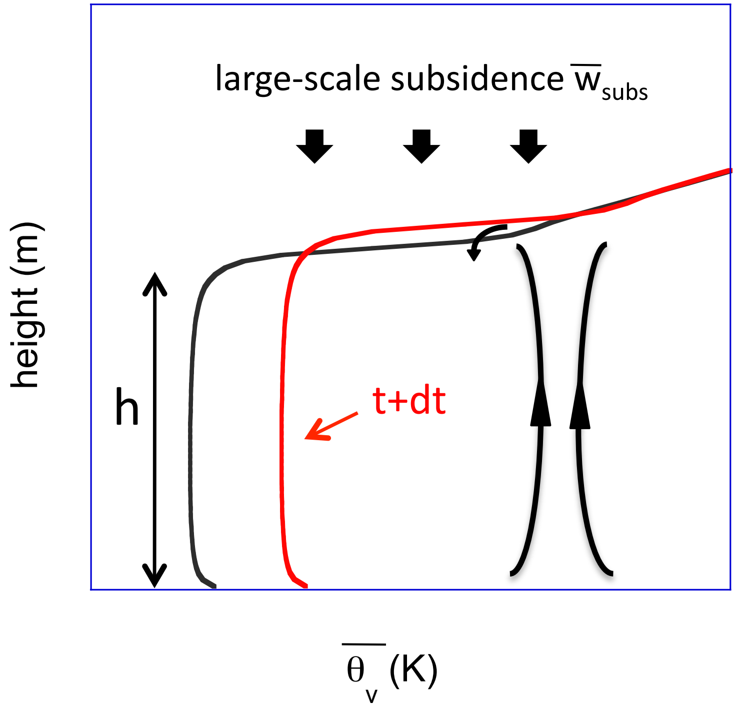
entrainment rate $w_e = \frac{dh}{dt}$

conservation equation

$$h\theta + \theta_{ft} dh = (h + dh)(\theta + d\theta)$$

$$\frac{d\theta}{dt} \cong \frac{dh}{dt}\frac{\Delta\theta}{h} = \frac{w_e \Delta\theta}{h}$$

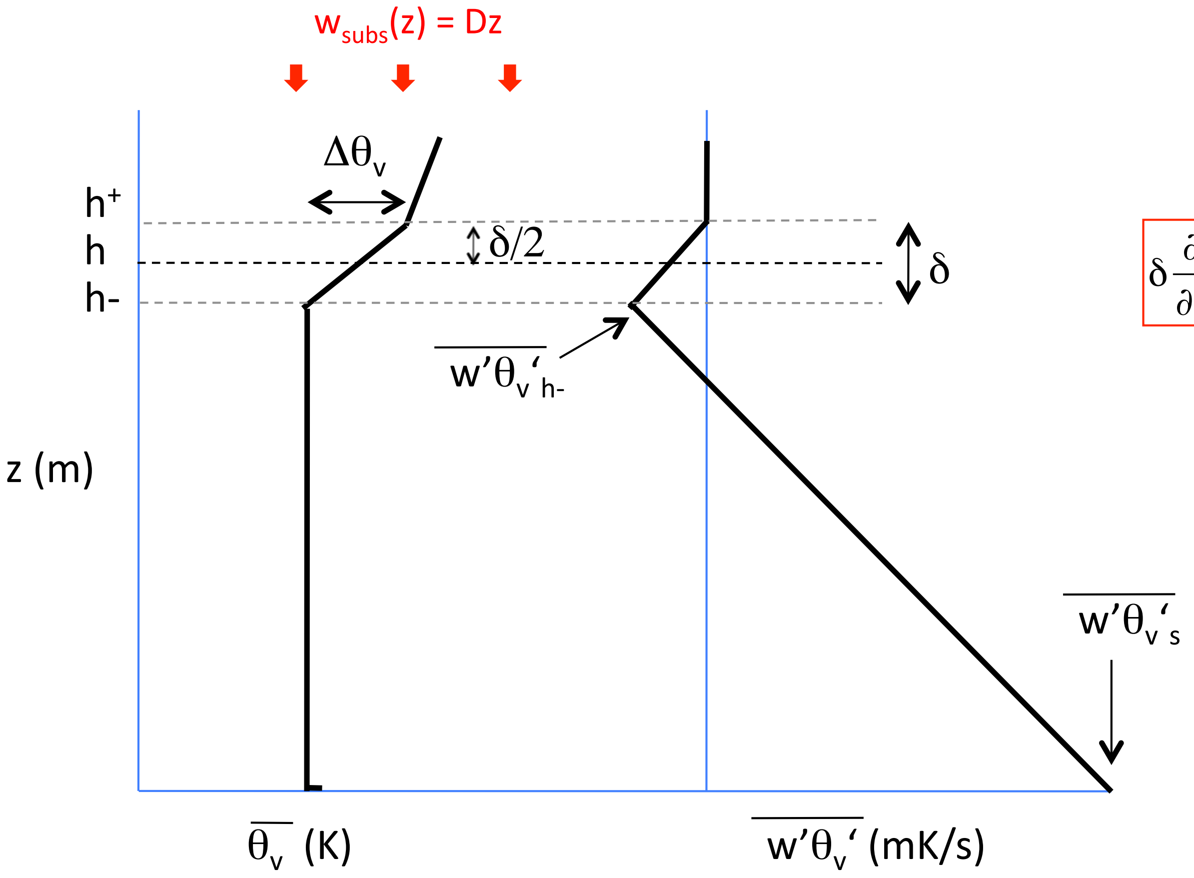
Budget equation for the boundary layer depth h including large-scale subsidence



$$\frac{dh}{dt} = w_e + \overline{w}_{subs}(z = h)$$

$$\frac{\partial \overline{w}_{subs}}{\partial z} = -\left(\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y}\right) \equiv D(z)$$

Schematic of the CBL: "the first-order jump model"



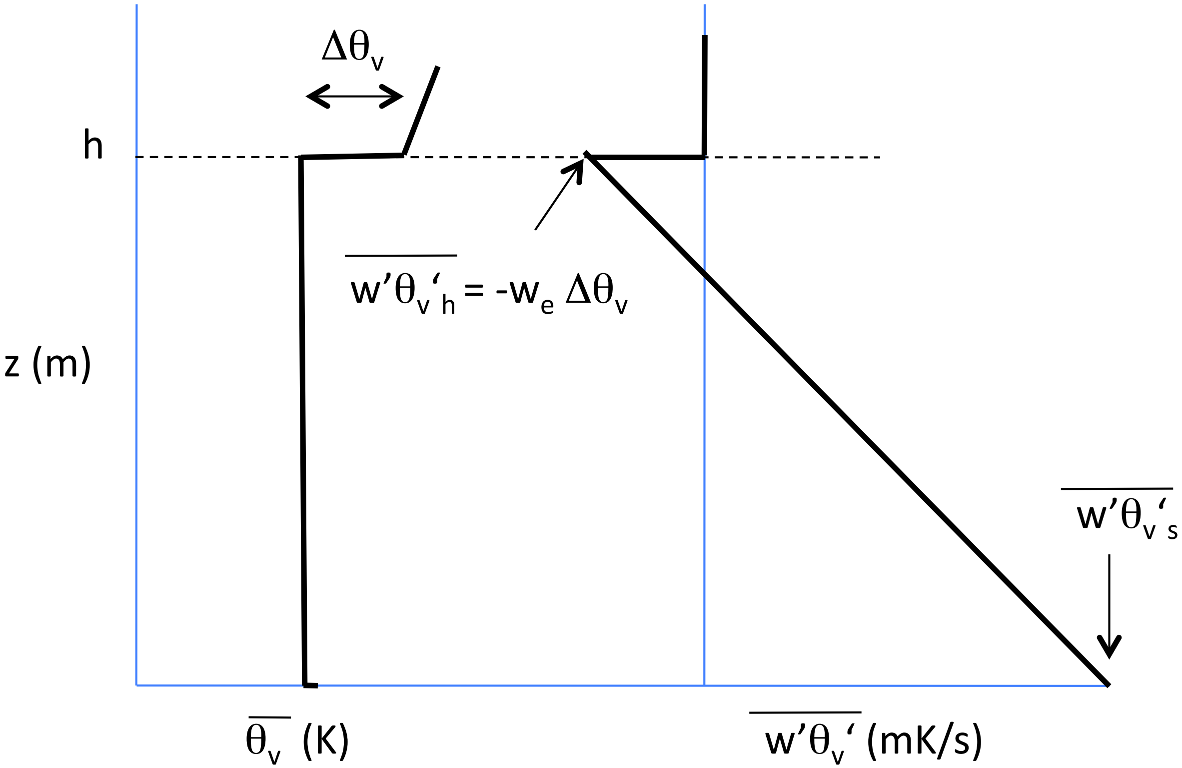
$$\frac{\partial \overline{\theta_v}}{\partial t} = -\frac{\partial \overline{w'\theta_v'}}{\partial z} - \overline{w}\frac{\partial \overline{\theta_v}}{\partial z}$$

$$\delta \frac{\partial}{\partial t}\left[\overline{\theta_v} + \frac{1}{2}\Delta\theta_v\right] = \Delta\theta_v \frac{\partial h^+}{\partial t} + \overline{w'\theta_v'}_{h^-} - w|_{h^- + \frac{\delta}{2}}\Delta\theta_v$$

$$h^- \frac{\partial \overline{\theta_v}}{\partial t} = -\overline{w'\theta_v'}_{h^-} + \overline{w'\theta_v'}_s$$

Van Zanten et al., 1999

Schematic of the CBL: The zeroth-order jump model



vertically integrated budget equation

$$h\frac{\partial\overline{\theta_v}}{\partial t} = w_e\Delta\theta_v + \overline{w'\theta_v'}_s$$

TKE (E), buoyancy fluxes and viscous dissipation

$$\frac{\partial \overline{E}}{\partial t} = \underbrace{\frac{g}{\theta_0}\overline{w'\theta_v'}}_{\text{buoyancy}} \underbrace{- \overline{u'w'}\frac{\partial \overline{U}}{\partial z} - \overline{v'w'}\frac{\partial \overline{V}}{\partial z}}_{\text{shear production}} \underbrace{- \frac{\partial \overline{w'E}}{\partial z} - \frac{1}{\overline{\rho}}\frac{\partial \overline{w'p'}}{\partial z}}_{\text{turbulent transport}} \underbrace{- \nu\overline{\left(\frac{\partial u_i'}{\partial x_j}\right)^2}}_{\text{viscous dissipation} = \varepsilon}$$

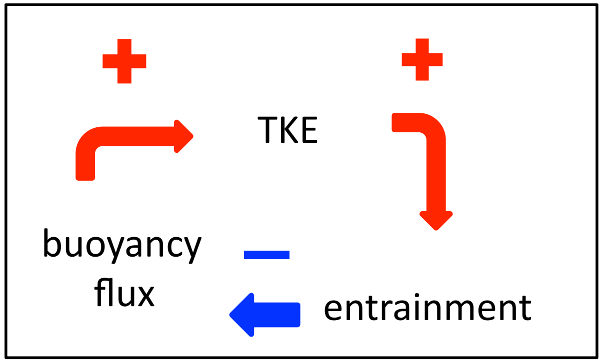
Steady-state vertically integrated TKE equation

- consider a situation without mean wind
- turbulent transport terms vanish
  (fluxes are zero at surface and just above the turbulent layer)

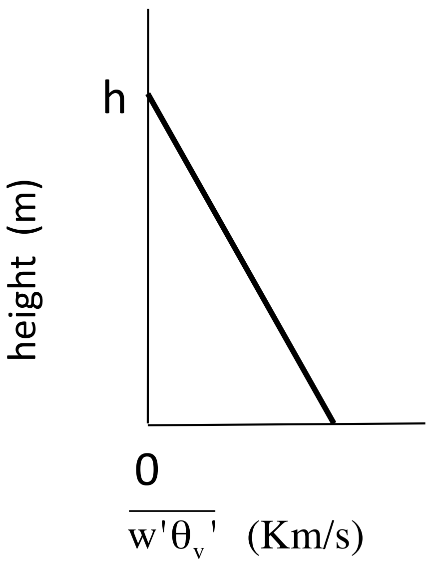
$$0 = \int_0^h \frac{g}{\theta_0}\overline{w'\theta_v'}dz - \int_0^h \varepsilon dz$$

How large is the entrainment rate?

$$\int_0^h \varepsilon\, dz = \int_0^h \frac{g}{\theta_0}\overline{w'\theta_v'}\,dz = \frac{1}{2}\frac{gh}{\theta_0}\left[\overline{w'\theta_v'}_s - w_e\Delta\overline{\theta_v}\right]$$

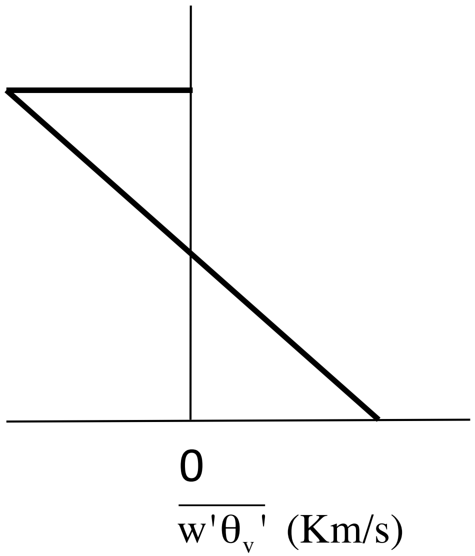


maximum TKE production



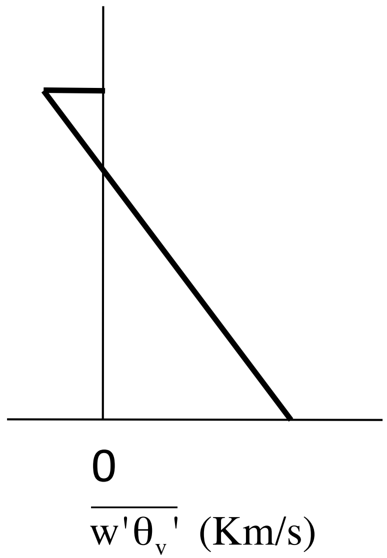
minimum (zero) TKE production

$$w_e\Delta\overline{\theta_v} = -\overline{w'\theta_v'}_s$$

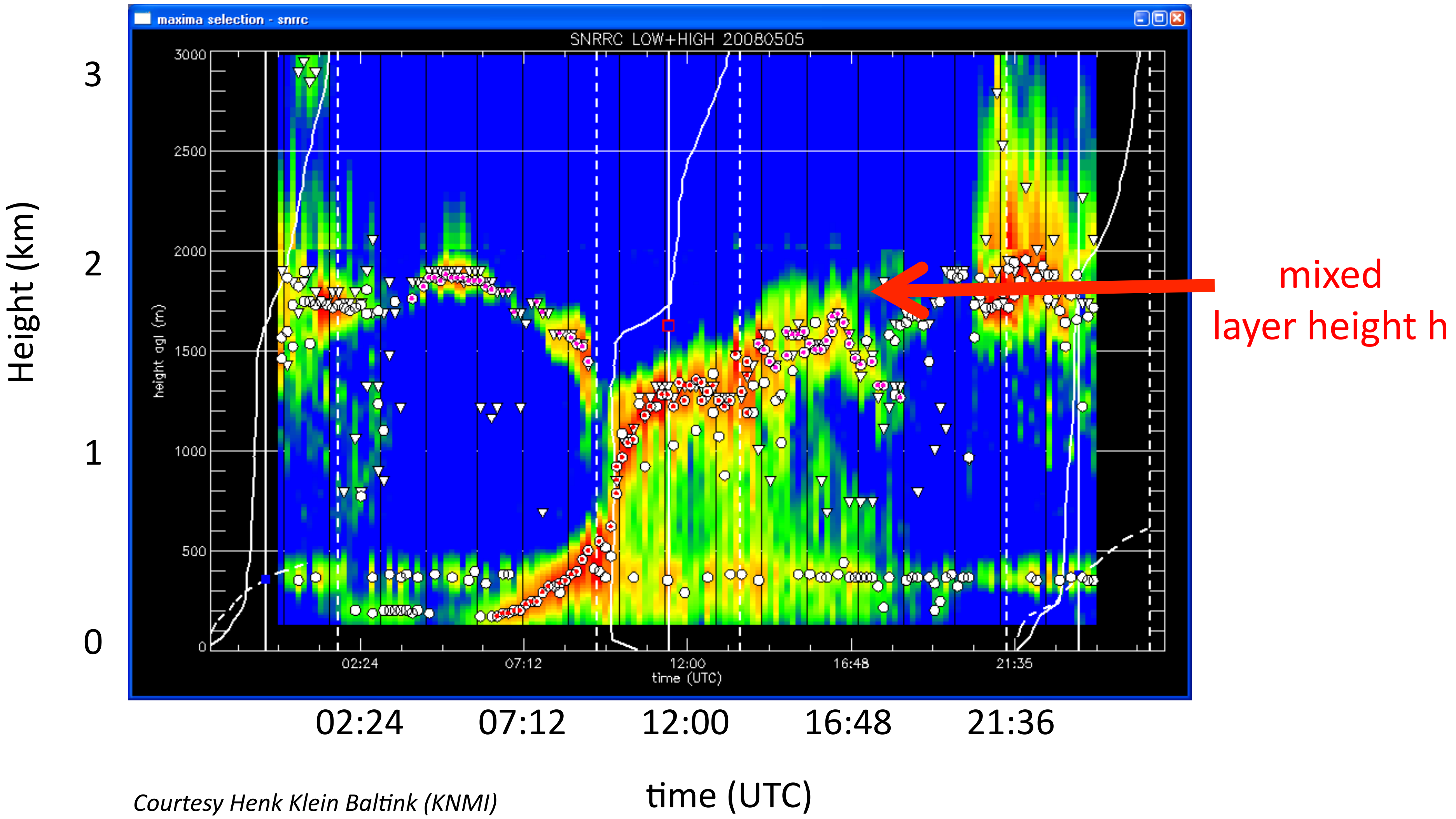


A≈0.2 (observations, LES)

$$w_e\Delta\overline{\theta_v} = -A\overline{w'\theta_v'}_s$$



Evolution of the convective boundary layer (Cabauw, The Netherlands)



Courtesy Henk Klein Baltink (KNMI)

Steady-state CBL height

$$\frac{\partial \overline{w}}{\partial z} = -\left(\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y}\right) \equiv D$$

assume D(z) = cst ⟶

$$\frac{\partial h}{\partial t} = w_e - Dh$$

equilibrium boundary layer height: $h_{eq} = \frac{w_e}{D} = \frac{A}{D} \frac{\overline{w'\theta_v'}_s}{\Delta\overline{\theta}_v}$

Shallow boundary layer depth if

- Strong large-scale divergence
- Small surface buoyancy flux
- Strong thermal inversion

Entrainment parameterization for the clear convective boundary layer

$$\frac{w_e}{w_*} = \frac{A}{Ri_{w_*}}$$

Relevant scales

Length scale h

Convective velocity scale $w_*^3 = \frac{gh}{\theta_0} \overline{w'\theta_v'}_0$

Non-dimensional stability parameter

convective Richardson number $Ri_{w_*} = \frac{gh}{\theta_0} \frac{\Delta\overline{\theta}_v}{w_*^2}$

On some Causes of the Formation of Anticyclonic Stratus as observed from Aeroplanes. (Proc. R. Soc., Edinburgh, vol. 37, 1917, pp. 137-148.)

The Upper Air: Some Impressions gained by Flying. (Edinburgh, J. Scot. Meteor. Soc., vol. 18 (ser. 3), 1918, pp. 3-12, pls. 8.) (Both papers by Captain C. K. M. Douglas, R.A.F.)

Also in Monthly Weather Review 1917

Capt. Douglas is to be congratulated on these two papers, and particularly on the photographs in the latter (four of which are reproduced). At the time the papers were published the information could not be given, but it may now be stated that the photographs and observations were taken from aeroplanes especially allotted for meteorological work by the Royal Air Force in France.

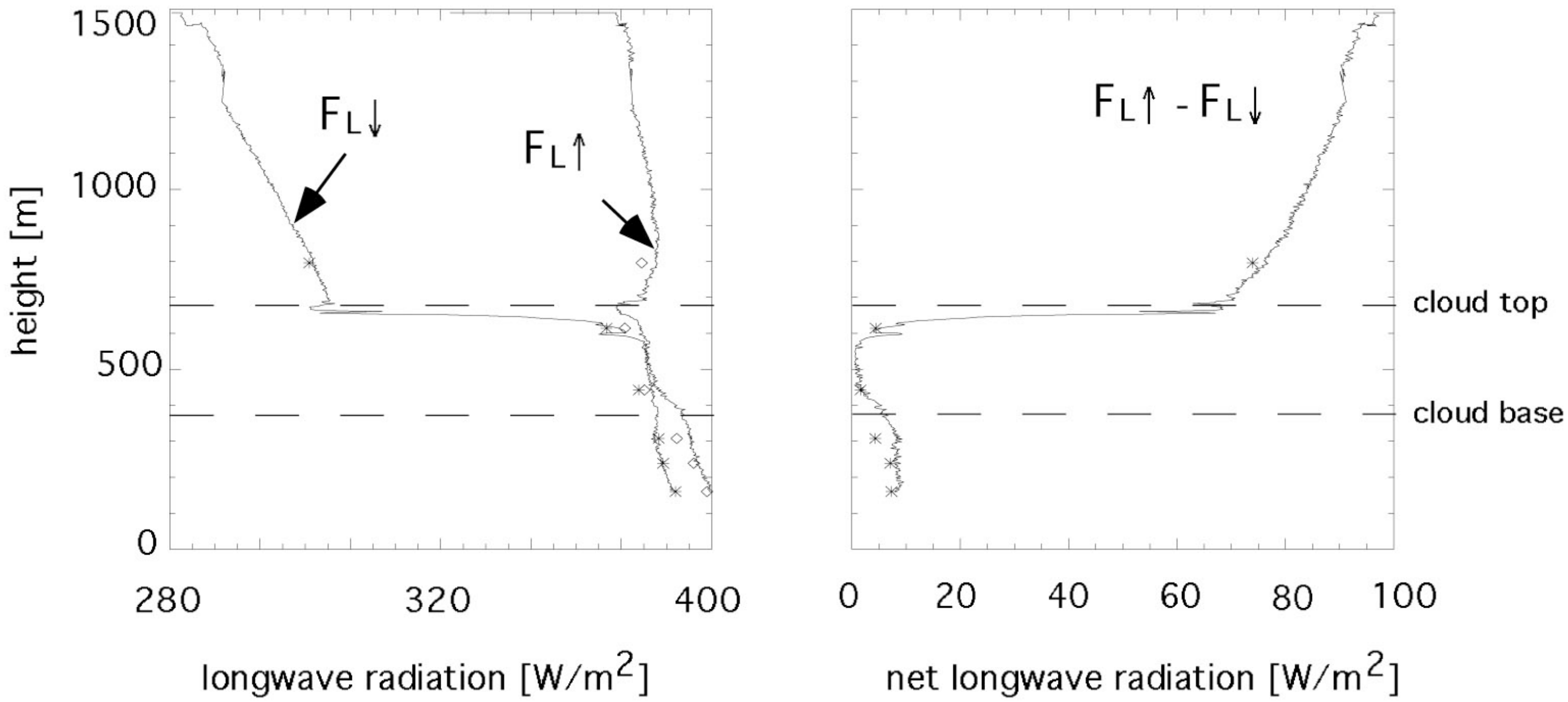
As a result of his observations Capt. Douglas states that stratus clouds have an adiabatic gradient below them and an inversion above, and that the same relation holds for well-defined layers of haze. He has found these clouds common on the northern and eastern sides of anticyclones and on the southern side in winter, and ascribes their formation to various causes.

In the second paper, which refers to observations in northern France in the spring and summer of 1918, these results are extended. The photograph Fig. 1 shows a typical sheet of strato-cumulus 1000 feet thick with its upper surface at about 7000 feet and an inversion of 7° F. above it. Capt. Douglas says that it is difficult to explain this cloud, that "all that can be said with confidence is that the air had not been disturbed by any considerable convectional disturbance for several days," and that "this seems to be one of the most important conditions for the development of cloud sheets with inversions above them, and helps to explain their prevalence in anticyclonic conditions."

The reviewer is inclined to ascribe much of the stratus and strato-cumulus of anticyclonic conditions to direct outward radiation from the layer of moist air which starts the cloud, and subsequently to radiation from the cloud itself. The air of an anticyclone above the almost inevitable inversion is very dry and is therefore pervious to radiation. There may be objections to this theory, but if we assume that these clouds are formed by turbulence it must be remembered that turbulence requires wind to produce it and the clouds belong to the type of weather which produces calms and light winds.

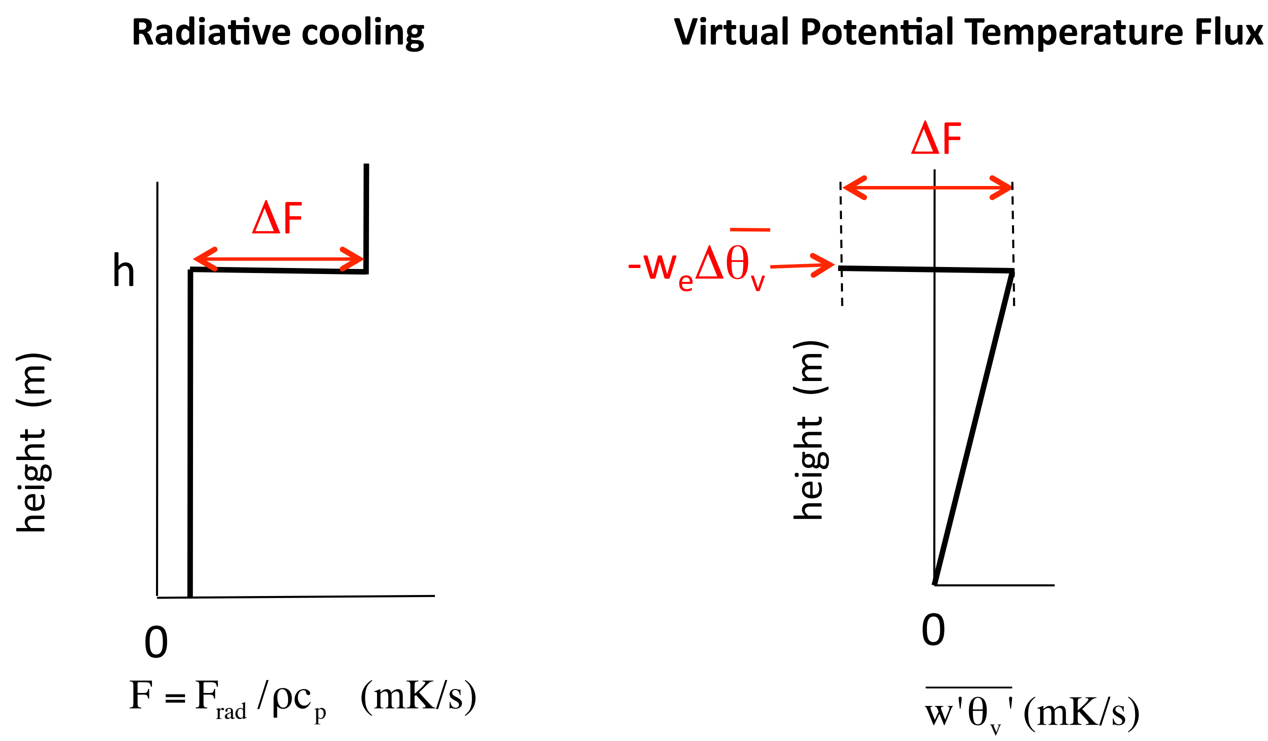
The details of some large inversions are given. At 2 p.m. on December 19, 1917, a rise of 12° F. occurred in the first 1600 feet. This is very unusual for the daytime, though common enough at night. At 10 a.m. on January 6, 1918, the surface was at 18° F., but at 1500 feet the temperature had risen to 46°, an inversion of 28° F. On this date the temperature at 4000 feet had risen 19° F. since the preceding day.

Longwave radiative cooling in stratocumulus



ASTEX Flight A209, Duynkerke et al. (1995)

Top-driven convection:
the smoke cloud ("stratocumulus" without latent heating effects)



Bretherton et al. (1999)

Non-dimensional entrainment parameterization

Convective velocity scale

$$w_* = \frac{gh}{\theta_0}\overline{w'\theta_v'}_0$$

modify to take into account forcings at different heights

$$w_*^3 = 2.5\int_0^h \frac{g}{\theta_0}\overline{w'\theta_v'}dz$$

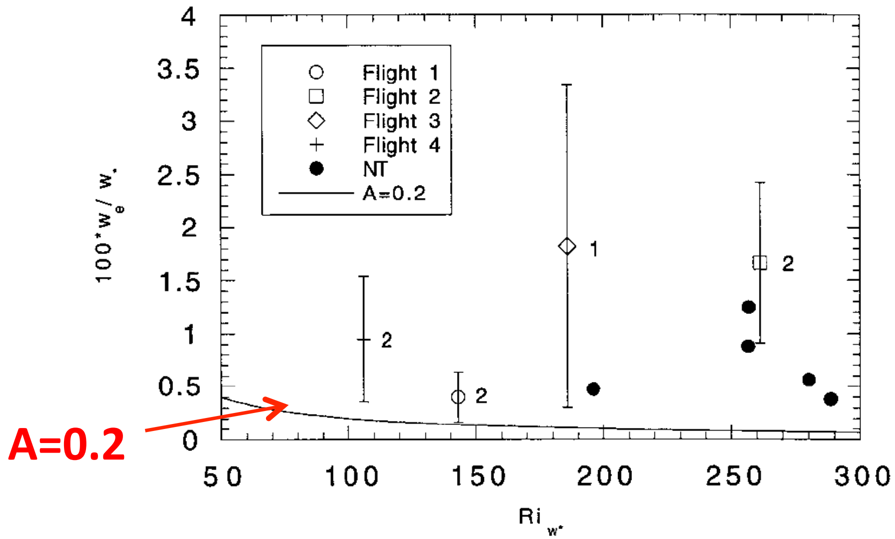
factor 2.5 to keep consistency with original CBL formulation

$$\frac{w_e}{w_*} = \frac{A}{Ri_{w_*}}$$

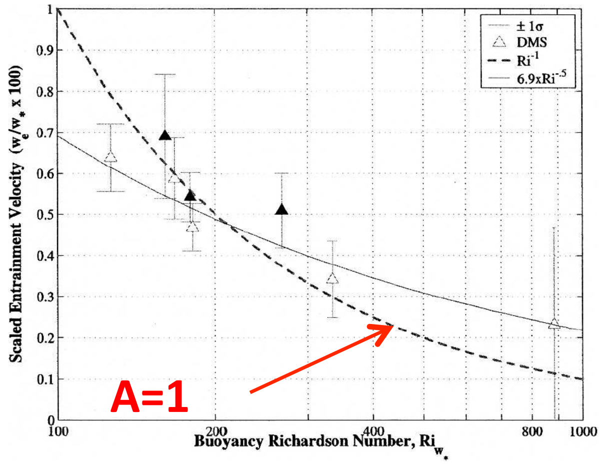
does it work for stratocumulus?

Entrainment rates in stratocumulus

$$\frac{w_e}{w_*} = \frac{A}{Ri_{w_*}}$$



De Roode and Duynkerke (1997)

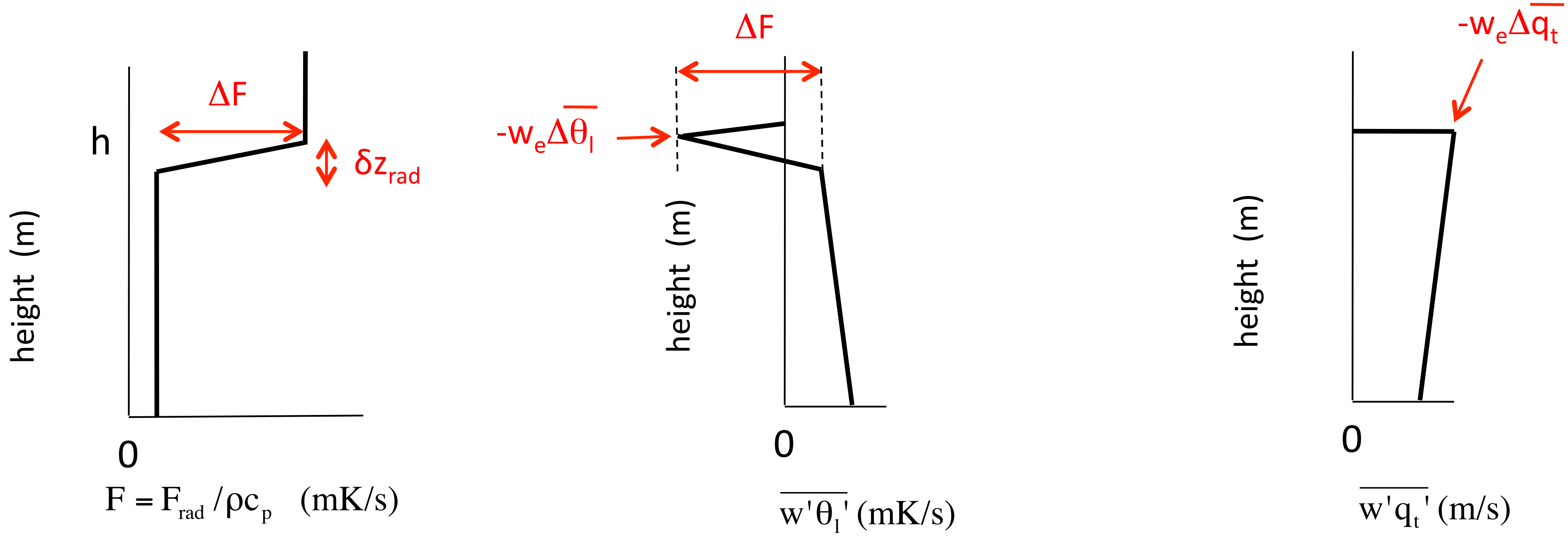


Faloona et al. (2005)

Fluxes of conserved variables $\phi = \{\theta_l , q_t\}$

Quasi steady-state

$$\frac{\partial}{\partial z}\frac{\partial \bar{\phi}}{\partial t} = -\frac{\partial}{\partial z}\left[\frac{\partial \overline{w'\phi'}}{\partial z} + \frac{\partial \bar{F}_{\phi,\text{source}}}{\partial z}\right] = 0 \quad \Longrightarrow \quad \overline{w'\phi'}(z) + \bar{F}_{\phi,\text{source}}(z) = az + b$$



Express θ_v' as a function of θ_l' and q_t'

Conserved variables $\theta_l = \theta - \frac{L_v}{c_p \Pi} q_l$ $q_t = q_v + q_l$

Definition $\theta_v = \theta\left(1 + \varepsilon_I q_v - q_l\right)$ $\varepsilon_I = \frac{1}{\varepsilon} - 1$, $\varepsilon = \frac{R_d}{R_v}$

Perturbation for dry case $\theta_v' = \left(1 + \varepsilon_I \overline{q_v}\right)\theta' + \varepsilon_I \overline{\theta} q_v'$

↓

$$\theta_v' = A_d \theta_l' + B_d q_t'$$

$$A_d = 1 + \varepsilon_I \overline{q_v} \approx 1.01 \quad , \quad B_d = \varepsilon_I \overline{\theta} \approx 180 \text{ K}$$

Saturated case, $q_l > 0$

Perturbation θ_v

$$\theta_v' = \theta'\left(1 + \varepsilon_I \overline{q_v} - \overline{q_l}\right) + \overline{\theta}\left(\varepsilon_I q_v' - q_l'\right)$$

Clausius-Clapeyron (CC)

$$\frac{dq_s}{dT} = \frac{L_v}{R_v}\frac{q_s}{T^2} \quad \Rightarrow \quad q_s' = q_v' = \gamma T' \approx \gamma\theta' \quad , \quad \gamma \equiv \frac{L_v}{R_v}\frac{\overline{q_s}}{\overline{T}^2}$$

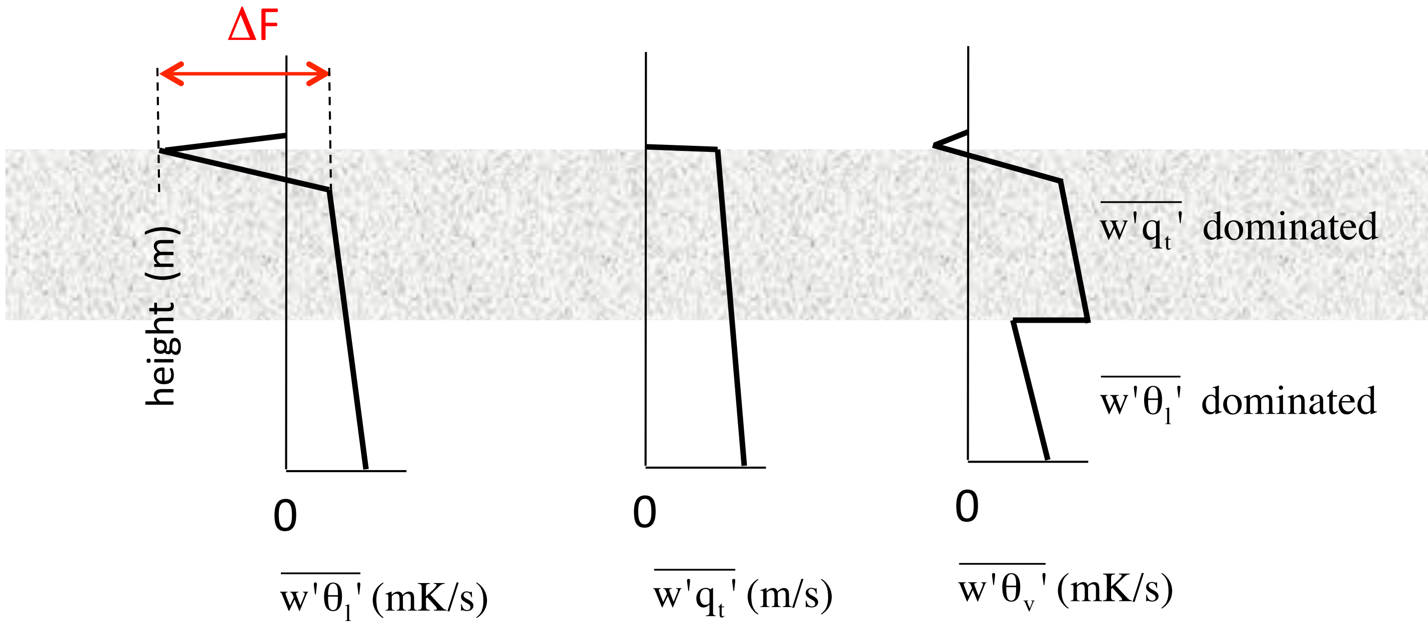
θ_l' and CC

$$\left(1 + \frac{L_v}{c_p \Pi}\gamma\right)\theta' = \theta_l' + \frac{L_v}{c_p \Pi} q_t'$$

$$\theta_v' = A_m \theta_l' + B_m q_t'$$

$$A_m = \frac{1 - \overline{q_t} + \left(\overline{q_v} + \gamma\overline{\theta}\right)/\varepsilon}{1 + \dfrac{L_v}{c_p \Pi}\gamma} \quad , \quad B_m = \frac{L_v}{c_p \Pi} A_m - \overline{\theta}$$

Fluxes of conserved variables $\phi = \{\theta_l , q_t\}$

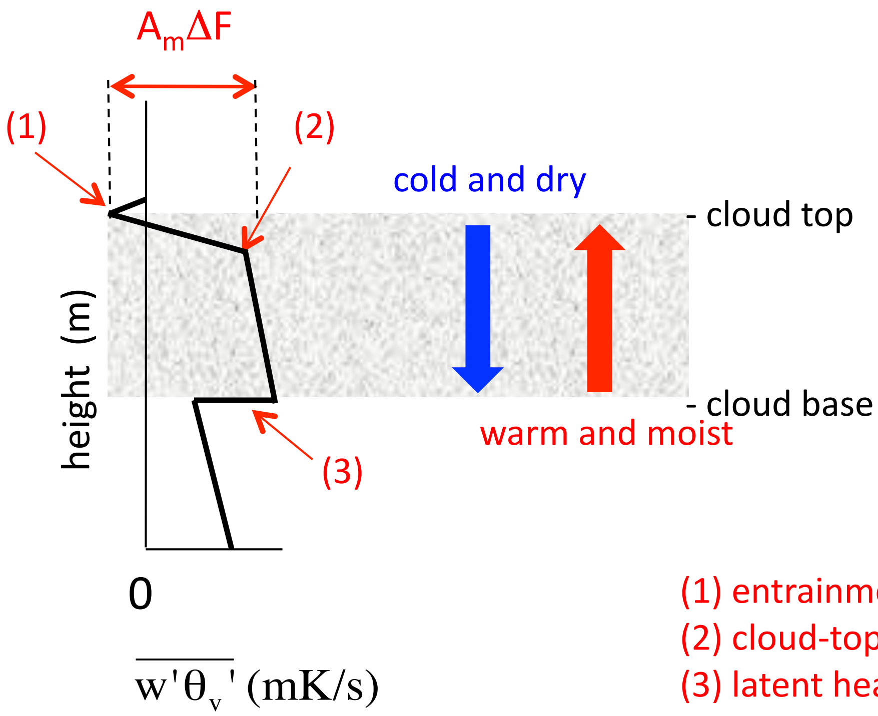


$$\overline{w'\theta_v'} = A_{d,m}\,\overline{w'\theta_l'} + B_{d,m}\,\overline{w'q_t'}$$

$ql = 0$, $A_d = 1.01$ $\quad$ $B_d = 170$ K

$ql > 0$, $A_m = 0.55$ $\quad$ $B_m = 1100$ K $\quad$ for T = 281 K, p = 980 hPa

Stratocumulus buoyancy flux



(1) entrainment (including effect of radiative cooling)
(2) cloud-top radiative cooling
(3) latent heating (condensation)